Economics 113 - Winter 2009 Professor Spearot
Introduction to Econometrics
Name John P. Answer Key

Midterm Exam # 1

The exam is closed book and closed notes. Please show your work step by step. Simple calculators may be used (no graphing or financial calculators and no cell phones)

You must show your work to receive full credit

Problem 1 (15 points)
You have four kids. They currently measure 5, 4, 4, and 3 ft tall. Further, their ages are 14, 10, 9, and 3, respectively.

a. Calculate the mean height and age. (3 points)

$$\hat{\mu}_H = \frac{5+4+4+3}{4} = \underline{4} \quad +1$$

+1 for work

$$\hat{\mu}_A = \frac{14+10+9+3}{4} = \underline{9} \quad +1$$

b. What is the covariance between height and age? (3 points)

$$\hat{\sigma}_{HA} = \frac{1}{n-1}\sum_{i=1}^{n}(H_i - \hat{\mu}_H)(A_i - \hat{\mu}_A) \quad +1$$

work

$$= \frac{1}{4-1}\left((5-4)(14-9) + (4-4)(10-9) + (4-4)(9-9) + (3-4)(3-9)\right)$$

$$= \frac{1}{3}\left(1 \cdot 5 + 0 \cdot 1 + 0 \cdot 0 + (-1)(-6)\right) \quad +1$$

$$\boxed{\hat{\sigma}_{HA} = \frac{11}{3} = 3.667} \quad +1$$

c. Congratulations, you've had another child. The new mean height all five kids is 3.4 ft. What is the height of your newborn? (4 points)

$$3.4 = \frac{5+4+4+3+x}{5} \qquad +2 \text{ for setup}$$

$$17 = \underbrace{5+4+4+3}_{16} + x$$

$$\Rightarrow \quad x = 1 \quad \left(\boxed{\text{New Height} = 1} \right)$$

+2 for answer

d. Using the original four kids, suppose that each grow older by one year, but their heights do not change. What is the effect of a one year increase in age on covariance? (5 points)

There is no change since adding a constant to each variable doesn't affect dispersion. +5 or nothing if no work

Or calculate New $\hat{\mu}_H = 5 + 1$

$$\hat{\sigma}_{HK} = \frac{1}{3}\Big((6-5)(14-9) + (5-5)(10-9) + (5-5)(10-10) + (4-5)(4-10)\Big) \quad +2$$

$$= \frac{1}{3}\big((1)\cdot 5 + (-1)(-6)\big)$$

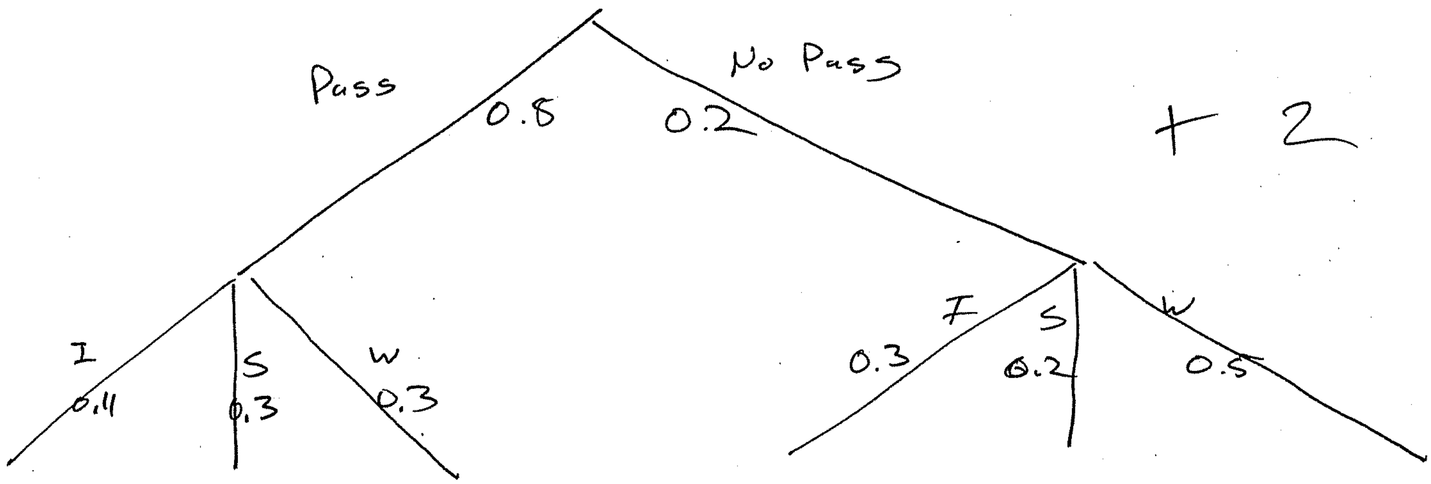
$$\boxed{\hat{\sigma}_{HA} = \frac{11}{3} = 3.67} \quad +2$$

Problem 2 (10 Points)

Our economy is in deep, deep trouble. President Obama is planning a huge stimulus package, which will either pass through the Congress, or not. The probability that his stimulus package passes is 0.8.

Of course, it is not certain that the package will work. Suppose that given a stimulus package, the economy improves with probability 0.4, stays the same with probability 0.3, and worsens with probability 0.3. In contrast, if there is no stimulus package, the economy improves with probability 0.3, stays the same with probability 0.2, and worsens with probability 0.5.

Given that the economy worsens, what is the probability that the Obama stimulus package passed through Congress?



+ 2

Find $\Pr(\text{Pass} \mid W) = \dfrac{\Pr(\text{Pass} \cap W)}{\Pr(W)}$ + 2

$$\Pr(\text{Pass} \cap W) = \Pr(\text{Pass}) \cdot \Pr(W \mid \text{Pass})$$

$$= 0.8 \cdot 0.3 = \underline{0.24}$$ + 2

$$\Pr(W) = \Pr(\text{Pass} \cap W) + \Pr(\text{No Pass} \cap W)$$

$$= 0.24 + (0.2)(0.5) = \underline{0.34}$$ + 2

$$\boxed{\Pr(\text{Pass} \mid W) = \frac{\Pr(\text{Pass} \cap W)}{\Pr(W)} = \frac{0.24}{0.34} = 0.706}$$ + 2

Problem 3 (15 Points)
The average number of games rolled by the dude per day is characterized by a normal distribution with mean 3 and standard deviation 1.5.

a. What is the probability that the dude rolls 4 games per day? (2 points)

0

+2

b. What is the probability that the dude rolls more than 4 games per day? (4 points)

$$Z_4 = \frac{4-3}{1.5} = \frac{1}{1.5} = \frac{2}{3} \quad +1$$

$$\underbrace{Pr(R \geq 4) = Pr\left(Z > \frac{2}{3}\right)}_{+1} = \underbrace{1 - Pr\left(Z < \frac{2}{3}\right)}_{+1} = 1 - 0.7454 = \boxed{0.2546} \quad +1$$

c. What is the probability that the dude rolls between 1 and 2 games per day? (4 points)

$$Z_1 = \frac{1-3}{1.5} = -\frac{2}{1.5} = -\frac{4}{3} \qquad Z_2 = -\frac{2}{3} \quad +1$$

$$\underbrace{Pr(1 \leq R \leq 2) = Pr\left(-\frac{4}{3} \leq Z \leq -\frac{2}{3}\right)}_{+1} = Pr\left(Z < -\frac{2}{3}\right) - Pr\left(Z < -\frac{4}{3}\right) \quad +1$$

$$= 1 - Pr\left(Z < \frac{2}{3}\right) - \left(1 - Pr\left(Z < \frac{4}{3}\right)\right)$$

$$= (0.2546) - (1 - 0.9082)$$

$$\boxed{Pr(1 \leq R \leq 2) = 0.1628} \quad +1$$

d. Suppose that along with games bowled, white russians consumed per day follows a uniform distribution [0,10]. Assuming that games bowled and white russians consumed are independent from one another, what is the probability of the dude rolling between 3 and 4 games OR consuming between 2 and 5 white russians per day? (5 points)

$Z_3 = \frac{3-3}{1.5} = 0$

$Pr(3 \le R \le 4) = Pr\left(0 \le Z \le \frac{2}{3}\right)$

$= Pr\left(Z < \frac{2}{3}\right) - Pr(Z < 0)$ +1

$= 0.7454 - 0.5 =$

$\boxed{Pr(3 \le R \le 4) = 0.2454}$

$Pr(2 < W < 5) = \frac{5-2}{10-0} = \frac{3}{10}$ +1

+2

$Pr((3 \le R \le 4) \cup (2 < W < 5)) =$

by indep

$Pr(3 \le R \le 4) + Pr(2 \le W \le 5) - Pr(3 \le R \le 4) \cdot Pr(2 \le W \le 5)$

$= 0.2454 + \frac{3}{10} - 0.2454 \cdot \frac{3}{10}$

$= \boxed{0.47178}$ +1

Extra Credit: Recall from class that the uniform distribution has the probability density function $f(x) = \frac{1}{d-c}$, where d is the upper bound of the distribution and c is the lower bound. Please prove that this distribution satisfies the two basic properties of a continuous distribution. (5 points)

Since $d > c$, $f(x) > 0$ for all x

+2

$$\int_c^d \frac{1}{d-c}\,dx = \frac{1}{d-c}\int_c^d dx = \frac{1}{d-c}\cdot(d-c)$$

$$= 1$$

+3

$$\int_{x \in \Omega} f(x)\,dx = 1$$